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SYMPOSIUM

Using Active Learning to Teach Concepts and Methods in Quantitative Biology

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Synopsis This article provides a summary of the ideas discussed at the 2015 Annual Meeting of the Society for Integrative and Comparative Biology society-wide symposium on Leading Students and Faculty to Quantitative Biology through Active Learning. It also includes a brief review of the recent advancements in incorporating active learning approaches into quantitative biology classrooms. We begin with an overview of recent literature that shows that active learning can improve students' outcomes in Science, Technology, Engineering and Math Education disciplines. We then discuss how this approach can be particularly useful when teaching topics in quantitative biology. Next, we describe some of the recent initiatives to develop hands-on activities in quantitative biology at both the graduate and the undergraduate levels. Throughout the article we provide resources for educators who wish to integrate active learning and technology into their classrooms.

Introduction

Why active learning?

Over the past 15 years, many studies on science education have demonstrated the success of active learning when compared with passive, lecture-based learning at the college level in science and mathematics (e.g., Springer et al. 1997; Hake 1998; Handelsman et al. 2004; Smith et al. 2009, 2011; Epstein 2013; Code et al. 2014; Ellis et al. 2014; Freeman et al. 2014; Linton et al. 2014a). Interactive teaching styles, focusing on conceptual learning, hands-on activities, and discussion with the opportunity for immediate feedback have been shown to be significantly more effective than traditional lecture-based courses using a variety of metrics (e.g., Springer et al. 1997; Handelsman et al. 2004; Ellis et al. 2014; Freeman et al. 2014). Specifically, problem-based programs have been successful in improving conceptual learning, problem-solving ability, retention of content, and students' satisfaction over traditional passive-learning formats, such as lectures, in a wide variety of both science major and non-major courses (Hmelo-Silver 2004; Anderson et al. 2011; Welsh 2012; Ellis et al. 2014; Linton et al. 2014a, 2014b). Furthermore, several studies have shown that small-group, inquiry-based instruction is particularly effective for populations of women and minority students (Cooper and Robinson 1998; Beichner 2008; Haak et al. 2011; Freeman et al. 2011).

As a result of this research, several national organizations have called for a shift in the way undergraduates are educated in biology and mathematics from traditional lectures to student-centered, active-learning strategies (APBI Taskforce 2008; AAAS 2009; AAMC–HHMI Committee 2009; National Research Council 2003; Steen 2005a, 2005b; Jungck and Marsteller 2010; Labov et al. 2010; Jungck and Schaefer 2011; Epstein 2013; Ledder et al. 2013; Herreid et al. 2014; Hodgen et al. 2014; Anguelov and Markov 2014). These reports note that active-learning pedagogy represents the best way to teach fundamental concepts and ways of thinking in biology and mathematics that will help to make American university students successful in research, industry, and medicine.

Why use active learning in quantitative biology?

Quantitative biology applies quantitative techniques to advance understanding of biological problems. Mathematical modeling is a key tool in understanding these problems, in which descriptive or predictive models are described that focus on capturing the interactions of the driving processes and influences of a biological system, rather than on capturing patterns in the data gathered from observing or measuring the system. Although quantitative biology includes statistical and informatics-based approaches, mathematical modeling is a cornerstone of many courses in quantitative biology.

Compared with traditional mathematics courses in which students must master established concepts and problem-solving techniques, courses in quantitative biology may be more open-ended. Creativity and consideration of multiple approaches are hallmarks of higher-level mathematics, but many undergraduates have minimal exposure to mathematics in which there is more than one "right" answer. In courses that teach quantitative biology through the development of mathematical models, students are challenged to explore problems with many possible solutions and to develop evaluative skills through rigorous comparison of mathematical results to real systems. In addition, mathematical modeling lends itself well to inquiry-based, collaborative-learning activities that are inclusive of students with a range of strengths and academic backgrounds. Thus, biologically-motivated, model-building activities are a useful approach to the implementation of active learning techniques.

Purpose of this article

Implementing active learning strategies in the classroom is not always straightforward, and implementation requires planning to be successful. The term "active learning" can refer to instructional methods ranging from clicker systems to collaborative, team-based learning, to integrating undergraduate research experiences into a course. Instructors should establish well-defined student learning outcomes and carefully assess the effectiveness of these strategies (AAAS 2009). Without careful consideration of student learning outcomes and thoughtful implementation, active learning can result in minimal to no improvement in learning and retention by students (Andrews et al. 2011).

The major goals of this article are: (1) to introduce faculty to some recent initiatives and programs for developing active learning within quantitative biology; (2) to introduce faculty who want to incorporate active-learning strategies in their instruction to some of the resources needed to do so effectively; and (3) to identify a few areas that the authors feel could benefit from continued development within quantitative biology.

Initiatives in quantitative biology

Quantitative biology can be roughly defined as any area of biology that requires significant connections to mathematics, computer science, data science, physical sciences, and/or statistics. Interest in quantitative biology has been growing, reflected by an increase in specialized programs and courses at both the undergraduate and graduate levels (Olena 2014). This overview describes the range of approaches taken in quantitative biology education and the challenges associated with cross-training students in historically disparate fields. Our emphasis is on programs that are tied to mathematical concepts and skills rather than to those with mostly a computational or informatics emphasis.

Undergraduate and graduate programs

The Society for Mathematical Biology maintains a list of degree programs in Mathematical Biology that currently includes 14 major programs and 3 minor programs at the undergraduate level and over 35 programs at the graduate level at universities in the United States, as well as in Belgium, Canada, and the United Kingdom (Society of Mathematical Biology 2015). This list focuses on programs in mathematical biology and does not include the range of programs in computational biology and biological informatics that have also been proliferating rapidly over the past two decades. Since quantitative biology covers a broad collection of fields, challenges inherent in designing these programs echo the challenges of the quantitative biology classroom. How do we tailor a program to encompass a student population with diverse backgrounds and interests? How do we provide adequate content in both mathematics and biology? How do we fit all of this material into a limited time frame or sequence of courses? Finally, how do we link these formal educational experiences with students' experiences in the laboratory or field, particularly in an environment of expanding opportunities for undergraduate research?

Existing graduate programs have addressed these questions through an array of approaches. (Information on specific programs is available on the Society for Mathematical Biology education column, SMB survey.) Admissions requirements range from evidence of interest in quantitative and biological fields to a preference for those who had pursued double majors as undergraduates. Some programs require coursework in both quantitative courses and biology courses, while others use co-advising from other departments, rotations in experimental laboratories, and/or targeted interdisciplinary coursework to address these needs. A specific challenge in designing course requirements for these programs is that different areas of biology may require different quantitative skills. In addition, faculty from different primary departments and disciplines may have varying expectations regarding the content and format of programs. To address these issues, many programs maintain flexibility in their requirements to individualize programs to a student's research interests and future career goals.

Although interdisciplinary training provides novel opportunities for graduates working in industry, national laboratories, research institutes, and major medical centers, opportunities within academia may be more limited. Indeed, interdisciplinary programs have observed significant numbers of their graduates working outside academia (Society of Mathematical Biology 2015), and challenges with the recruitment and hiring of mathematical biologists in mathematics departments have been noted (Reed 2004).

Specialized courses in quantitative biology

In addition to institutions with degree programs in quantitative biology, many institutions offer specialized courses in this area. Typically, these courses are targeted at either biology or mathematics students. For example, students planning to enter careers in the health sciences (see also Section 5.3) can take an entry-level course, such as calculus, with a biological slant (see Ledder 2008; Neuhauser 2010; Adler 2012) or a more broadly based course that includes discrete methods as well as calculus (Bodine et al. 2014). Alternatively, a traditional calculus course may be replaced by a course in mathematical modeling that may be more immediately relevant to students in the life sciences (Eager et al. 2014). Upper-division courses focused on particular biological fields with strong quantitative connections, such as population ecology (Hastings 1996), field biology (Kokko 2007), and epidemiology (Keeling and Rohani 2007), represent another option that is regularly available.

Interestingly, some quantitative biology courses have been very successful in teaching and exploring mathematical biology at a high level through carefully selected teams of students with complementary backgrounds and skills (Karsai et al. 2011; Full et al. 2015). By working in groups that have been intentionally designed in this way, students are able to hone collaborative skills, an integral part of an interdisciplinary education, while tackling high-level problems in quantitative biology. Such group projects may be a component of upper-division modeling courses as well. Future work is needed to establish the feasibility of broader dissemination of courses that are completely group-project based.

Non-majors courses

In addition to teaching quantitative biology to mathematics, statistics, biology, or other Science, Technology, Engineering and Math Education (STEM) majors, there have been recent efforts to teach such topics to non-majors. For example, first-year seminars have grown in popularity since the turn of the century as a mechanism to improve academic achievement, retention, and student bonding and collaboration (Hyers and Joslin 1998; Tinto 1999; Starke et al. 2001). Most of these seminars are open to all undergraduates and have few, if any, prerequisites. Although there are some resources available for teaching courses to non-majors (Jungck 2012a; Jungck and Roy 2014), materials in quantitative biology for first-year seminars and other courses for non-majors are in great need of development since many available texts assume a working knowledge of calculus or beyond.

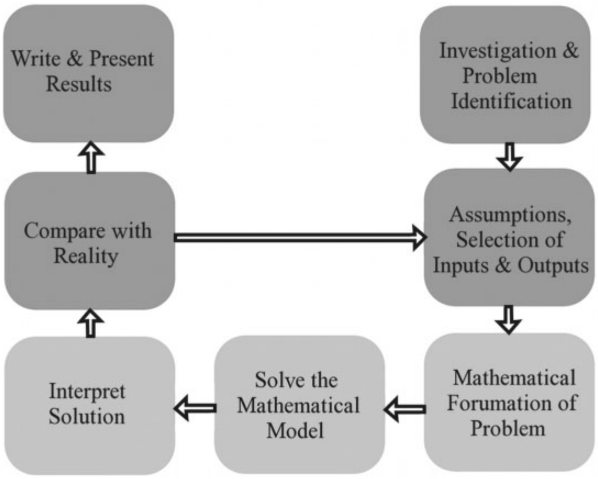

Fig. 1 Diagram showing the iterative process of mathematical modeling. Many courses in mathematical modeling and mathematical biology focus only on the bottom red part of the diagram.

Mathematical modeling in biology

Mathematical modeling is an integral component of quantitative biology. Although the entire process of developing and testing a mathematical model is inherently active (see Fig. 1), the process often is presented to students in a relatively passive format. Morris (1967) noted that many texts focus on examples of models that have already been developed, and the descriptions of such models usually involve justification rather than inquiry. In other words, the final model "works," and the text explains why it works. In such a structured approach, students often miss the key discovery aspects of model formulation and the evaluative aspects of comparing model solutions with reality. By changing the way students interact with modeling, we can provide lessons that engage content, provide realistic experiences, and develop distinctive skills (Jungck 2012b). A large number of the standard texts used at both the graduate and undergraduate level in mathematical biology are designed to expose readers to models, not to help them develop their modeling capabilities. Teaching about models is not the same as teaching about modeling (SIAM–NSF Working Group 2012).

Active learning through model building

The formulation of a mathematical model requires understanding of both the biology of the problem and the modeling approach being implemented, and guiding students through this process can be challenging (Niss 2012). Galbraith (1989) summarized the following three approaches to teaching mathematical modeling:

- Generalized applications approach: The instructor introduces the model and the students manipulate it under controlled conditions.
- Structured modeling approach: The students are exposed to all stages of the modeling process, but the instructor exerts considerable control over the mathematical model that is to be used.
- Open modeling approach: All stages of the modeling process are completed with limited assistance from the instructor. Students study a problem at the level of mathematics they are comfortable using.

Each of these approaches helps students to develop distinct skills, and instructors can scaffold these approaches to facilitate the development of mathematically mature modeling skills. The generalized applications approach may provide a useful format to introduce students to modeling techniques and canonical models in mathematical biology. The structured modeling approach allows students to develop model-building skills in an environment that maximizes successful learning outcomes. Finally, the open modeling approach provides students with realistic model-building experiences and encourages creative solving of problems.

Alternatively, the structured modeling approach can be applied on a case-by-case basis throughout the model-building exercise, depending upon the progress of each group. Galbraith (1989) outlined three types of intervention that can be used:

- Subtle intervention: The instructor subtly suggests which model to use.
- Open intervention: After the students have written down their own models, the instructor then presents the model that is commonly used.
- Delayed intervention: The instructor allows the class to complete the modeling process on their own, and then presents the model commonly used.

By maintaining awareness of the types of intervention appropriate for a given situation, the instructor can tailor students' interactions to maximize exposure to realistic model building while ensuring a baseline level of progress.

How training in model-building benefits students

Courses with an emphasis on model building enhance students' performance in quantitative thinking, and they also help to develop other skills. Since model building requires a diverse set of skills, construction of original mathematical models for new biological problems challenges all students at all levels. Mathematical modeling tends to level the mathematical playing field: every student from the upper level mathematics major to the pre-medical student with one semester of Calculus can make substantial contributions to modeling projects. Due to the highly interdisciplinary nature of model-building in quantitative biology, students both in mathematics and in biology must often leave their disciplinary comfort zones to work across disciplines. This exercise poses additional challenges, and provides specific educational benefits, to each subset of students.

Students in mathematics

Students in mathematics are exposed to mathematical models in many classes, but their experiences of true model building may be limited. Traditional mathematics classes are typically focused on solving given equations. In more applied classes, there may be some discussion on formulating problems and on interpreting solutions. However, modeling in these classes often is presented as an established endpoint (e.g., the simple harmonic oscillator or the wave equation) rather than an open-ended problem. Therefore, providing opportunities for students to grapple with the formulation and assessment of original models in quantitative biology teaches skills that may not be addressed in other parts of the undergraduate mathematics curriculum.

Furthermore, the contrasts between the disciplines of mathematics and biology encourage learning. In mathematics, there is an emphasis on the discovery of underlying truths. By contrast, biology involves unavoidable uncertainty, and students quickly learn to question results. Such questioning is a key precept of biology, and the exposure to hypothesis-driven inquiry is vital for mathematics students. Mathematicians may be unaccustomed to presenting their work in terms of questions or hypotheses, and this may be a strong disadvantage for students in mathematics who are crossing disciplinary boundaries or interacting with researchers in other scientific fields through collaborations or cross-disciplinary competitions for grants or fellowships. Course work that teaches students to frame their efforts in terms of hypotheses may also benefit students in fields such as computer science and theoretical statistics.

Model building also provides mathematics students with more experience working on real-world applications, working in teams, and communicating to non-mathematicians. These outcomes are particularly significant when one considers a recent survey of industrial managers performed by the Society for Industrial and Applied Mathematics (SIAM–NSF Working Group 2012). When asked about key strengths for industrial mathematicians, managers noted the following:

- Understanding of and interest in practical applications (41%).
- Communication skills, interaction with others (36%).
- Breadth of knowledge of other areas (23%).

Although these are necessary skills for most professions, developing these skills is particularly valuable for the many undergraduate math majors who will join the industrial workforce.

Students in biology

Biology has a reputation for having less quantitative emphasis than other sciences such as physics and chemistry (Fawcett and Higginson 2012). However, most current biological research requires a strong quantitative background that may not be provided in a standard undergraduate biology curriculum (Bialek and Botstein 2004; Gross et al. 2004; Speth et al. 2010; Feser et al. 2013; Ledder et al. 2013). Model-building courses provide exposure to mathematics in a format that motivates and engages students through relevant biological questions. In addition to developing quantitative skills, these courses demonstrate how modeling can be used both as interpretive and investigative complements to experiments.

A single course in quantitative biology may not equip biology students to model their own experimental results, but it can develop skills that enable students to engage with mathematical modeling at a more sophisticated level. Students achieve a familiarity with mathematical modeling that will allow them, as future researchers, to communicate with mathematical collaborators and to critically evaluate mathematical modeling approaches by others.

Initiatives/resources in active learning

A large body of resources exists for implementing active-learning strategies in college-level classrooms. However, typical instructors may not be familiar with these resources. In this section, we present several types of resources that are useful for instructors hoping to use these pedagogies for the first time or build upon previous efforts to enhance students' learning. These resources, their references, and website addresses are summarized in Table 1.

Table 1 Summary of initiatives and resources facilitating active learning of quantitative biology

Name	Description	Website or reference
Initiatives in quantitative biology		
Courses for majors	Calculus	Adler (2012), Neuhauser (2010), Ledder (2008)
	Broader mathematics	Bodine et al. (2014)
	Mathematical modeling	Eager et al. (2014), Galbraith (1989)
	Upper-level biology	Hastings (1996), Kokko (2007), Keeling and Rohani (2007)
	Quantitative biology	Full et al. (2015)
Courses for non-majors	First-year seminars	Hyers and Joslin (1998), Starke et al. (2001), Tinto (1999)
	Quantitative biology	Jungck and Roy (2014), Jungck (2012a)
Resources in active learning		
Collaborative learning	Books and papers	Barkley et al. (2005), Linton et al. (2014a), Full et al. (2015), Joshi et al. (2007)
	SCALEUP	Beichner (2008), NCSU (2011)
Technology		
On-line software	Appsbar	http://www.appsbar.com/
	MathWork's Desktop and Web Deployment tool	http://www.mathworks.com/desktop-web-deployment/deploying-code-web-application.html
	R Studio's Shiny	http://shiny.rstudio.com/
In-class	Clickers/Personal Response Systems	Gauci et al. (2009), Greer and Heaney (2004), Smith et al. (2011), Andrews et al. (2011)
	Poll Everywhere	http://www.polleverywhere.com
	Interactive Whiteboards	Dhindsa and Shahrizal Emran (2010, 2011)
Social media	Twitter	Drew (2015); http://twitter.com
	Figshare	http://figshare.com
Specialized software	Numb3r5 Count Project	http://bioquest.org/numberscount/
	Biological ESTEEM Project	http://bioquest.org/esteem/
	Netlogo	https://ccl.northwestern.edu/netlogo/
	PhET Interactive Simulations	http://phet.colorado.edu
	SimBio	http://simbio.com
	Cell Collective	Helikar (2012, 2015)
Class activities		
Hands-on labs	BioMathLab Project	Kohler et al. (2010), Powell et al. (2012), Haefner (2008)
	Textbooks	Vogel (1996, 2013), Cornette (2012), Robeva et al. (2008); Mahaffy (2005), Keller and Thompson (2012a, 2012b, 2012c)
Other activity repositories	QUBEShub	http://qubeshub.org/
	National Center for Case Study Teaching in Science	https://sciencecases.lib.buffalo.edu/cs/collection/
	University of Tennessee, Knoxville	http://www.tiem.utk.edu/~gross/bioed/modulelist.html
Workshops	NIMBios	http://www.nimbios.org/
	BioQUEST	http://bioquest.org
	Symposium on BEER	https://about.illinoisstate.edu/biomath/beer
Evaluation and assessment		
In-class instructor	Peer Review	Falchikov and Goldfinch (2000)
	RTOP	Sawada et al. (2002), Smith et al. (2013)
	TDOP	Smith et al. (2013), Hora and Ferrare (2013), http://tdop.wceruw.org/Document/TDOP-Users-Guide.pdf
	COPUS	Smith et al. (2013)
Course evaluation	TPI	Wieman and Gilbert (2014)

Vision and change—a place to start

For instructors interested in integrating active-learning strategies into their courses, an excellent place to start is the Vision and Change document, a report from the American Association for the Advancement of Science 2009 conference on undergraduate biology education (AAAS 2009). Vision and Change includes a list of student-centered learning resources and a list of assessment instruments and instructional methods with supporting references. Vision and Change also identifies resources for integrating multiple forms of assessment for tracking students' learning and provides methodology for using the information gathered to improve the classroom environment (AAAS 2009).

Resources for developing active-learning techniques

Several books and articles present practical information about learning and incorporating active-learning strategies. Some of these resources present basic classroom techniques for creating a student-centered learning environment (Jungck 1991; Asokanthan 1997; Fortus et al. 2004; Barkley et al. 2005; Handelsman et al. 2007; Tanner 2013; Aikens and Dolan 2014). Others focus on creating learning environments for more specific conceptual material, such as experimental design (Brownell et al. 2013). Freeman et al. (2011) described how to increase course structure from low (lecture-based) to high (active learning), and a follow up study by Eddy and Hogan (2014) included some specific examples of how course structure was improved for an introductory biology class. Other articles provide overviews on how technology can be used to incorporate active learning in the classroom. Some examples include the use of interactive whiteboards (Dhindsa and Shahrizal Emran 2010, 2011) and personal-response systems (Greer and Heaney 2004; Gauci et al. 2009).

Collaborative learning

Collaborative learning environments give students the opportunity to engage and explore conceptual material with other students and instructors during lectures, laboratory-based activities, or projects outside of class time. Working in peer groups significantly improves students' comprehension, independent of the skill of the instructor (Springer et al. 1997; Linton et al. 2014a, 2014b). Importantly, significant improvements from collaborative discussion came primarily in higher-level conceptual material, application, and synthesis, all of which are critical skills for students of quantitative biology (Linton et al. 2014a). Furthermore, collaborative learning environments more accurately mimic the environments in which students will eventually contribute as workers in industry or in academia.

Collaborative learning spans a great number of techniques (summarized by Barkley et al. 2005) that can easily be incorporated into traditional college courses. Many techniques can be added to existing lecture-based courses for in-class discussion (i.e., turn to your neighbor and discuss) or paired with in-class technologies such as clickers (see the "In-class technology" section) for rapid feedback. More traditional laboratory-based courses can also benefit from techniques that increase students' engagement with instructors and peers without changing existing laboratory exercises.

Student groups can be self-formed or instructor assigned, ephemeral (turn to your neighbor) or long-lasting (semester-long projects). When students are allowed to form groups, these groups tend to be composed of students of the same major, intellectual background, and interests. Working in such a homogeneous group can create more competition than cooperation where ideas from similar backgrounds compete for the group's use. In a mathematical biology course, instructors can assign group members with complementary backgrounds and skills to provide depth in areas than would otherwise be inaccessible to a homogenous group of mathematicians or biologists. This strategy has been used successfully with multi-disciplinary topics such as comparative biomechanics (Full et al. 2015) and mathematical modeling to increase the depth of material covered and the creativity and sophistication of students' work, and there are many examples from the National Science Foundation (NSF) program Interdisciplinary Training for Undergraduates in Biological and Mathematical Sciences (UBM) that paired mathematics and biology undergraduates in research projects (Joshi et al. 2007).

A well-studied example of a successful project that implemented an effective interactive learning environment in large introductory science courses is the SCALEUP Project that was piloted at North Carolina State University (Beichner 2008; NCSU 2011). SCALEUP was designed to replace the common laboratory/lecture scheme with an integrated collaborative, group-based classroom environment. In a SCALEUP classroom, students are divided into groups of 3, and they work through activities that are interspersed throughout the class. An important outcome of this strategy is increased communication among students and between students and their instructors. Significant evaluation of the project shows that these approaches positively influence students' learning. Specifically, problem-solving skills are improved, conceptual understanding is enhanced, students' attitudes are improved, failure rates are reduced, and there is better retention of "at risk" students. More than 50 schools across the country have adapted SCALEUP with the goal of getting students to work together to investigate interesting problems and to increase interaction with instructors.

Barkley et al. (2005) is an excellent resource for implementing all aspects of group work in a college-level setting. It covers group introductions, strategies for structuring, and evaluating group work through a large variety of Collaborative Learning Techniques. It also covers troubleshooting for common pitfalls such as problems with inequitable work, cheating, and students' resistance.

Technology

Pedagogical approaches that employ technology can be very useful for incorporating active learning into different classroom settings. The scalability of many of these technological tools allows faculty to combine the individual involvement possible in smaller classes with the resources of large classes. For example, tools such as clickers and polls can facilitate participation by students, even in large lectures, while social media can facilitate discussions and extend classroom interactions to engage the greater public.

Online software and application tools

Many mathematical models involve computing projects that may exceed the coding ability of undergraduate students. Creating online applications for participants is a good way of bridging this gap by making computational models more accessible both to students and instructors. Several services aid in the creation and dissemination of applications. Appsbar (http://www.appsbar.com/) aids in creating applications for different platforms. MathWork's Desktop and Web Deployment tool (http://www.mathworks.com/desktop-web-deployment/deploying-code-web-application.html) creates MATLAB-based components for use on the Web and does not require additional software for the end user to operate the application. Similarly, Shiny is a free, open-source package in R (R Studio, http://shiny.rstudio.com/) that allows users to build web-based applications directly from R.

In-class technology

Clickers are an increasingly popular method of integrating existing lecture-based courses with inquiry-based strategies (Smith et al. 2011). Clickers are small devices purchased by universities or students that allow students to answer multiple-choice questions posed to them by lecturers. Answers by students can be displayed as they come in real time or as a summary at the end of a set period of time. Many groups have reported improved learning when clickers are used in lecture-based introductory courses, but success tends to be tied heavily to specific implementations and to the skill of the lecturer (Andrews et al. 2011).

An alternative to clickers, Poll Everywhere allows students to use their computers or cell phones to answer multiple-choice or short-answer questions during class (http://www.polleverywhere.com/). Poll Everywhere allows instructors to create, track, and grade answers to polls. As with clicker-based polls, these polls may be used to track results in real-time or to evaluate learning over time. Many universities have professional accounts with Poll Everywhere, making it easy for instructors to sign up and get help with the tools of the website.

Social media

Social media provides a unique way to engage students in material and increase the diversity of voices in the classroom (for more information, see Drew 2015). Services such as Twitter (http://twitter.com), a micro-blogging platform, help to create, develop, and publish ideas, as well as promote engagement with material both from students and the public (Darling et al. 2013). Other platforms such as FigShare (http://figshare.com/) provide a way to make content more accessible to students off-campus (Drew 2015). Additionally, social media is an effective way to build equality in the classroom and to promote increased diversity within science by providing a greater exposure to a wider array of voices and experiences (Drew 2015).

Specialized software projects

The Numbers Count Project is dedicated to the use of quantitative tools for solving biological problems. This initiative was led by Claudia Neuhauser at the University of Minnesota and was funded by the Howard Hughes Medical Institute. The project website, http://bioquest.org/numberscount/, includes a variety of open resources including biological data, introductory mathematical modules for biology and chemistry, statistics modules, resources from workshops, course materials for calculus and introductory statistics, and a variety of other resources and tools.

The Biological ESTEEM project provides Excel simulations and tools for exploring experimentation with mathematical models in the life sciences (http://bioquest.org/esteem/). This effort is led by Anton Weisston, John Jungck, and Raina Robeva, and is supported by NSF and the Mathematical Association of America. Excel was chosen as the general development environment given its availability and familiarity to most students and faculty. Currently, the project's site includes over 40 modules in subject areas ranging from pharmacokinetics to island biogeography to models of continuous growth. New modules are continually being developed, and all are welcome to contribute their own materials using the specifications provided on the project's site.

The agent-based modeling tool Netlogo (https://ccl.northwestern.edu/netlogo/) has been specifically developed for education about models that follow the actions of individual agents. This approach has been applied in essentially every area of biology and the pedagogy for its use in educational settings has been very well documented. The software is freely available, as well as implemented through a web-interface, and can be used with real-time interactions that allow students to jointly and/or individually modify components of the model, such as a disease simulation in which students can control the movements of individuals so as to prevent spread of infection.

Additionally, several other software projects are available to aid in teaching biological systems through manipulating simulations. These include PhET Interactive Simulations (http://phet.colorado.edu), SimBio (http://simbio.com), and the Cell Collective (Helikar 2012, 2015).

Published activities of the class

One barrier to implementing active-learning strategies in college-level courses is the investment of time required to stray from previously used materials and develop new activities. However, several resources exist to lessen this initial commitment of time by providing a place to share exercises, laboratories, activities, and advice on structuring the class. Below, we highlight a few examples of these resources as places to start.

Published hands-on quantitative biology laboratories

Experimental laboratories that incorporate mathematical modeling allow students to obtain their own data for validating the model and to consider the assumptions that are made during the modeling process. Exercises that combine experiment and theory are often used in physics and engineering laboratory courses, but they are less common in the life sciences. The development of wet laboratories that connect to biomathematical modeling has the potential to increase the retention of mathematics by introducing mathematics within the context of biological systems using discovery-based approaches.

The BioMathLab Project at Utah State University was aimed at creating quantitative laboratory experiences in the biology curriculum. Several papers that describe specific laboratory activities that are easy and inexpensive to implement have been published as a result of this effort. Kohler et al. (2010) described an activity in which students can compare the movement of brine shrimp to a diffusion model. The diffusion coefficient is estimated for individual brine shrimp, and the diffusion equation with this coefficient is then used to predict the distribution of many brine shrimp in a petri dish. Powell et al. (2012) described a set of activities that encourage students to create their own models of flow from a leaky bucket. Additional laboratories may be found on the BioMathLab website (Haefner 2008) and include activities related to osmosis, photosynthesis, cooling, optimal foraging, enzyme kinetics, and birds' flight.

In addition to resources for individual laboratories, there are also a variety of textbooks from both the mathematics and biological sciences that include hands-on activities and exercises and incorporate mathematical modeling. Vogel (1996, 2013) include hands-on activities and demonstrations that illustrate the importance of mechanical models in biology. Cornette (2012) developed a ''Wet-Lab'' Calculus for the Life Sciences. The associated laboratories include topics ranging from exponential growth to Fick's law to how crickets' chirp-rates depend on temperature. Three of the laboratories are now included in the National Council of Teachers of Mathematics website, Illuminations (Keller and Thompson 2012a, 2012b, 2012c). Virtual laboratories can also offer students an opportunity for discovery. Robeva et al. (2008) include simulations and animations that permit students to interact with biological processes and to perform virtual dissections. Mahaffy (2005) includes computer laboratories for simulating biological processes that complement a calculus course targeted to biologists.

Quantitative Undergraduate Biology Education and Synthesis

The Quantitative Undergraduate Biology Education and Synthesis (QUBES) project is funded by the NSF, and the QUBES consortium (http://qubeshub.org/) is an alliance of societies, institutions, and programs united to strengthen education in quantitative biology. The broad goals of the QUBES project are to (1) coordinate educational efforts in quantitative biology across disparate communities; (2) support faculty who wish to implement specific quantitative concepts and approaches to teaching; (3) increase the visibility, utility, and adoption of existing quantitative materials; and (4) track faculty's contributions to education in quantitative biology, and determine the features of QUBES that increase the success of implementation. The QUBES website (https://qubeshub.org/) contains numerous resources related to education, research, and collaboration in quantitative biology, including data, teaching materials, and models. The website also includes a suite of interactive software tools that can be run directly from a web browser, allowing easy use of these educational tools across platforms. Some of the software available includes NetLogo, R Studio, QtOctave, and pplane.

Other repositories

The National Center for Case Study Teaching in Science maintains a peer-reviewed collection of case studies for teaching concepts in science (https://sciencecases.lib.buffalo.edu/cs/collection/). Case studies provide an easy, exercise-guided method for incorporating active-learning strategies into lecture or laboratory time. Within the current collection, there are several case studies focusing on mathematical or biological concepts, yet very few address topics in integrated mathematical biology. However, this collection offers an easy way to share existing lesson plans with other educators.

At the University of Tennessee, Knoxville, a set of over 50 modules were developed to accompany a typical introductory biology sequence. Links to these modules are available at http://www.tiem.utk.edu/~gross/bioed/modulelist.html. Each module has a standard format consisting of introducing a biological question, defining key variables and their units, identifying a relevant mathematical model and the associated data to parameterize it, carrying out some analysis of the model, and providing further questions for students to investigate either individually or in groups.

The University of Maryland has developed a website, MathBench Biology Modules at http://mathbench.umd.edu, that highlights the mathematical underpinnings of topics in an introductory biology course. At least 36 modules cover biological topics ranging from population dynamics to cellular processes as well as more general topics such as measurement and visualization. The modules include interactive activities, games, and questions. The activities also incorporate a wide range of mathematical topics including statistics, modeling, and difference equations.

Workshops for sharing resources in quantitative-biology education

In addition to published and online resources for quantitative-biology education, workshops provide important opportunities for dissemination of methods and materials. They also provide excellent networking opportunities for educators. These workshops are typically hosted at a rotating assortment of colleges, universities, and other institutions such as the Howard Hughes Medical Institute (HHMI; https://www.hhmi.org/). Other common venues include annual society conferences; for example, this symposium had a companion session offering hands-on practice with quantitative-biology modules in classrooms. HHMI and NSF in particular have generously supported numerous workshops to promote and disseminate instructional materials and pedagogical innovations in quantitative biology.

Workshops are advertised on several websites, including those hosted by QUBES (https://qubeshub.org/), the National Institute for Mathematical and Biological Synthesis (NIMBioS, http://www.nimbios.org/), BioQuest (http://bioquest.org), and the International Symposium on Biomathematics and Ecology Education and Research (BEER; https://about.illinoisstate.edu/biomath/beer).

Evaluation and assessment

Evaluation and feedback are important tools for improving course structure, pedagogy, as well as the performance and effectiveness of instructors. Here, we discuss metrics that can be used to evaluate the effectiveness of instructors and courses.

Metrics for in-class evaluation of instructors

We are all accustomed to being "reviewed" by our students at the end of the semester in an instructor/course evaluation. At most colleges and universities there is a great deal of time and effort put into crafting forms that give meaningful feedback. However, there are several important limitations to these common metrics for evaluation. First, typically, instructors do not receive feedback until after the conclusion of the course. Although many instructors choose to conduct midterm evaluations, these are less standardized across and within institutions. Second, students are not trained observers in the classroom. Even more involved approaches, such as interviews with students, reflect a limited perspective. To complement evaluations by students, peer observers may carry out a detailed assessment

(Falchikov and Goldfinch 2000). Such peer feedback may be particularly important for instructors who are experimenting with novel pedagogical approaches, especially when these approaches push students out of their traditional comfort zones.

Methods of evaluation by observation can yield constructive feedback on the classroom environment that can be used for research, evaluation of programs, development of faculty, and institutional assessment. A number of protocols for faculty observers have been developed with the objective of providing a common language and format in which to make useful and constructive observations and allow reliable comparisons across classrooms. The protocols range in the amount of preparation and training required of the observer (Brown et al. 2008).

A widely used observation protocol for observation called Reformed Teaching Observation Protocol (RTOP) was developed by the Evaluation Facilitation Group of the Arizona Collaborative for Excellence in the Preparation of Teachers. To implement this protocol, trained observers judge measures like engagement of students and effectiveness of instruction, using statements ranging from "not at all" to "to a great extent." Typically, observers participate in multi-day training, and the protocol has been used with proven reliability when implemented by trained observers (Sawada et al. 2002). However, this protocol is less reliable when used by un-trained or weakly trained observers; since judgments may be observer-dependent, a lack of reliability can be problematic for comparison across classrooms (Smith et al. 2013).

Another protocol for observation is called the Teaching Dimensions Observations Protocol (TDOP), which was specifically designed to address post-secondary non-laboratory courses (Hora and Ferrare 2013; Smith et al. 2013). This protocol was developed as part of the Culture, Cognition, and Evaluation of a study of STEM Higher Education Reform funded by the NSF's Reese Program. The protocol is comprised of six categories: teaching methods, pedagogical strategies, cognitive demand, student–teacher interactions, student-engagement, and instructional technology. Observers use codes to describe the classroom environment and make observations at 2-min intervals over the meeting of the class. For example, an observer could describe the teaching method being used as "SGW" meaning students are working in small groups (Hora and Ferrare 2013). Hora and Ferrare (2013) gave detailed instructions on best practices and implementing peer observations using TDOP in a User's Manual: http://tdop.wceruw.org/Document/TDOP-Users-Guide.pdf.

The Classroom Observation Protocol for Undergraduate STEM (COPUS) is a procedure for observing faculty to make reliable characterizations of how faculty and students are spending their time in STEM classrooms (Smith et al. 2013). It was developed by science education specialists at the University of British Columbia to eliminate judgment on the part of the observer as well as to shorten the amount of training required of the observer. Faculty observers complete a training session of 1.5 h that prepares them to use the protocol to document what students and instructors are spending their time doing in the classroom using specific codes, similar to the TDOP. For example, an observer could describe what the instructor is doing using the code "Lec" to denote lecturing, or "AnQ" to denote answering student questions. However, with COPUS, the number of categories in which observations are made is reduced to 2: what are the instructors doing and what are the students doing (Smith et al. 2013).

Establishing a culture of peer observation in a department can facilitate the essential cultural shift necessary to engage deeply with the pedagogical issues born from an active classroom (AAAS 2009). Identifying like-minded peers with whom to discuss and collaborate on classroom issues will lay the groundwork for creating a community of support in which instructors can experiment with active learning and thoughtfully assess pedagogical innovation (from Panel discussion at SICB).

Metrics for assessing courses

Information and feedback about teaching can also translate to effective evaluation of the course itself. Especially in courses that involve active learning or a research component, the teaching strategies faculty employ are tied to the content. The Teaching Practices Inventory (TPI) was designed to give a detailed characterization of the teaching practices used in STEM courses, and it includes a quantitative measure of the extent to which research-based teaching practices are used in a course (Wieman and Gilbert 2014). This inventory differs from the observation protocols described above in that faculty complete the inventory on their own (there is no observer) in about 10 min and with little opportunity for subjective judgment. The TPI captures the teaching practices that are used over a semester rather than merely a snap-shot of one or two lessons that are observed using the aforementioned protocols. The information contained in the individual inventories can help improve courses and teaching and help to increase consistency across a department. Identifying the practices that are being used can also help with future implementation of a research component or active-learning strategy in other courses.

Identification of needs

Despite the available resources for integrating active-learning pedagogies into college-level classrooms, some areas specific to quantitative biology need more resources to bring these techniques into mainstream use. Below are several areas in which resource-gaps and significant challenges still exist.

Numerical simulations

One common challenge in quantitative biology courses is to successfully teach biology students who range from mathematics/biology double majors to those who have only taken one semester of calculus several years earlier. Another challenge is to go beyond the presentation of simple toy models to the development and analysis of complex real-world problems.

A solution to both challenges is to teach students how to use computers to solve mathematical models. Doing so will allow the students to focus on the construction, interpretation, and validation of mathematical models rather than getting caught up in the details of finding the solutions by hand. Some caution should be taken, however, so as to avoid the misuse of computation. This approach should include some basic introduction to numerical analysis and programming as well as exercises to explore common issues that could produce incorrect results. For example, students could use Euler's method to solve a simple ordinary differential equation with a known solution. By comparing the size of the time step chosen to the amount of error in the solution, they could discover issues of convergence of the numerical solutions.

Students' attitudes represent another challenge in teaching the use of computation. Students less familiar with programming tend to resist the use of the software and writing their own code. Often, they assume that computational exercises represent additional work that will be difficult for them to complete. One approach to help them discover that the computer can be used to make their lives easier is to introduce a module that requires tedious calculations and then to write a simple program that does the calculations automatically. For example, in a module incorporated into a lesson on Brownian motion, students could be asked to calculate a random trajectory for 30 time-steps. They would then be guided through a simple program that calculates the trajectory automatically and graphs the result. They can experiment with modifying the program to be extended to more time-steps, multiple random walks, and a calculation of the average distance from the starting point. Such an exercise highlights the utility of the program for accomplishing things faster and more easily than doing all the work by hand (Rubinstein and Chor 2014).

Examples of relevance to premeds

Quantitative biology programs typically focus on educating future researchers, but they also represent a distinctive niche for students who are preparing to enter careers in the health sciences. Although pre-health majors/tracks are known for their extensive requirements, many of the science courses that are part of a standard pre-health curriculum overlap with science courses that are required in quantitative biology programs. Furthermore, basic courses in calculus and statistics may also satisfy requirements for majors both in pre-health and quantitative biology. By exploiting this overlap, quantitatively-oriented pre-health students can use a quantitative biology major to explore their interdisciplinary interests and distinguish themselves in competitive admissions to medical school.

Stand-alone quantitative biology courses or short-course sequences provide another alternative to contribute to the preparation of pre-health students who lack access to or sufficient interest in quantitative biology programs. There are educational resources for statistical analyses of biological data, and there are many excellent educational materials for mathematical biology that use intuitive and classical examples from ecology, evolution, and epidemiology. However, there is a shortage of materials that use modeling applied to the level of tissues or below to tackle current problems that are appropriate for sophomore-level biology students, a critical point for students considering health-related fields. One example of a highly quantitatively-focused basic text at this level is Phillips et al. (2012) that could readily be used for a basic course in cell biology for students with stronger quantitative backgrounds. Tailoring course-topics to address problems with clear applications to human health emphasizes their immediate relevance to the career-goals of pre-health students, thereby encouraging these students' interests in mathematical biology and computation. More generally, the inclusion of quantitative biology courses in a standard pre-health curriculum can provide unique training that complements coursework from other fields. Specifically, the process of model-building, particularly in a group setting that draws on varied disciplinary strengths of group members, develops skills in individual and team-oriented critical thinking that are necessary for modern medical practitioners.

Examples using less traditional areas of mathematics

The use of mathematical techniques to address biological questions has exploded over the past decade, and biologically-motivated questions are driving the creation of new mathematics (Cohen 2004; Hunter 2010). Current courses in mathematical modeling tend to emphasize standard models based on differential-equations, but much progress has been made in developing the modeling potential in many other fields of mathematics. For example, approaches common in graph theory are used widely in applications ranging from network theory and the relationship between structure and function of neuronal networks to systems biology and questions involving motifs and pattern-recognition. Topological approaches have been used to understand DNA underwinding, overwinding, knotting, and tangling (Deweese et al. 2008). A short review of algebraic methods applied to biological problems is given by Robeva and Laubenbacher (2009), and these authors make a strong case for incorporating algebra in the mathematical biology curriculum since the topic is more familiar than differential equations to most students and faculty. Several recent textbooks have included some of these mathematical approaches in the context of biological applications (Robeva et al. 2008; Segel and Edelstein-Keshet 2013; Robeva 2015), but additional methods and current examples of problems relevant to many fields of biology and mathematics are in need of development for the undergraduate curriculum.

In addition, educational modules that teach mathematicians to frame their work in terms of hypotheses are not currently available. It is likely that such modules would also be very useful for students in computer science and theoretical statistics. In particular, mathematicians often are not aware of the large body of work that supports the use of strong inference in biological research (Platt 1964). It may be especially useful to introduce this approach in contrast to the modeling approach and in the context of inductive versus deductive reasoning (Glass and Hall 2008). Other educational modules that may be useful for mathematicians studying biology include illustrations of biological variation, diversity, experimental design, and measurement-error.

Conclusions and recommendations

The above survey focuses on the range of approaches in quantitative biology education as well as presenting several key areas the authors have identified as being ripe for the development of new educational materials. One important area of need includes the development of lesson materials and textbooks on topics in quantitative biology for non-majors, including students outside of the STEM fields. Another key area for the development of curriculum involves the development of biological examples that require the use of non-traditional applied mathematics, including graph theory, topology, and algebra. Interestingly, these two areas are not necessarily independent. Many quantitative and mathematical biology texts focus on the application of differential equations, and to some extent linear algebra, to problems in biology. It can be particularly challenging to use these techniques in a diverse class where some students are adept at calculus and linear algebra while others have never seen this material. The application of methods from topology, graph theory, and algebra offers a unique opportunity to introduce new tools to a broad audience while keeping advanced mathematics students engaged.

As more instructors incorporate active-learning techniques into their classrooms, there will also be more opportunities to assess the power of these approaches. Although there is strong evidence that active learning assists learning generally, the explicit interdisciplinary context of quantitative biology has not been assessed relative to the capability to enhance development of mathematical concepts or the creative aspects of mathematical model-building. To date, there are few studies addressing the impact of the connection between mathematical and biological disciplines on the educational goals in the respective disciplines. Thus, although the advantages of integrated approaches resonate with instructors, there is little non-anecdotal evidence that inclusion of biological examples motivates undergraduates to more readily conceptualize and effectively use the quantitative tools discussed. Going forward, there is a clear need for collaboration with researchers in education to determine which of the approaches and types of courses discussed above are most effective in enhancing learning in quantitative biology.

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